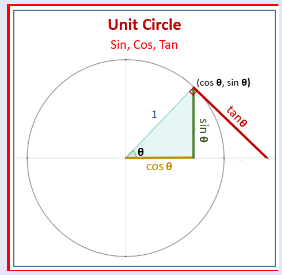
Trigonometry

Lecture 21



Feb 19-8:47 AM

Verify $\cos^2 x - \sin^2 x = \boxed{2\cos^2 x - 1}$

Recall

$\sin^2 x + \cos^2 x = 1$

$\sin^2 x = 1 - \cos^2 x$

$\cos^2 x - \sin^2 x =$

$\cos^2 x - (1 - \cos^2 x) =$

$\cos^2 x - 1 + \cos^2 x =$

$\boxed{2\cos^2 x - 1}$ ✓

Oct 3-10:28 AM

Simplify

$$\sin x \cos x \left(\tan x + \cot x \right)$$

$$= \sin x \cos x \tan x + \sin x \cos x \cot x$$

$$= \sin x \cancel{\cos x} \cdot \frac{\sin x}{\cancel{\cos x}} + \cancel{\sin x} \cos x \cdot \frac{\cos x}{\cancel{\sin x}}$$

$$= \sin^2 x + \cos^2 x = \boxed{1}$$

Oct 3-10:31 AM

Verify

$$\frac{1 + \tan^2 x}{1 - \tan^2 x} = \frac{1}{\cos^2 x - \sin^2 x}$$

$$\frac{1 + \tan^2 x}{1 - \tan^2 x} = \frac{1 + \frac{\sin^2 x}{\cos^2 x}}{1 - \frac{\sin^2 x}{\cos^2 x}} = \frac{\cos^2 x \cdot 1 + \cancel{\cos^2 x} \cdot \frac{\sin^2 x}{\cancel{\cos^2 x}}}{\cos^2 x \cdot 1 - \cancel{\cos^2 x} \cdot \frac{\sin^2 x}{\cancel{\cos^2 x}}}$$

$LCD = \cos^2 x$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x - \sin^2 x}$$

$$= \frac{1}{\cos^2 x - \sin^2 x}$$

Oct 3-10:34 AM

Verify

$$\tan^2 x - \sin^2 x = \tan^2 x \cdot \sin^2 x$$ ✓

$$\tan^2 x - \sin^2 x = \frac{\sin^2 x}{\cos^2 x} - \frac{\sin^2 x \cos^2 x}{\cos^2 x}$$

$$= \frac{\sin^2 x - \sin^2 x \cos^2 x}{\cos^2 x}$$

$$= \frac{\sin^2 x (1 - \cos^2 x)}{\cos^2 x}$$

$$= \frac{\sin^2 x}{\cos^2 x} \cdot (1 - \cos^2 x)$$

$$= \tan^2 x \cdot \sin^2 x$$ ✓

Oct 3-10:38 AM

Verify $$\frac{\sin^3 x + \cos^3 x}{\sin x + \cos x} = 1 - \sin x \cos x$$ ✓

Hint

$$A^3 + B^3 = (A+B)(A^2 - AB + B^2)$$

$$\frac{\sin^3 x + \cos^3 x}{\sin x + \cos x} = \frac{\cancel{(\sin x + \cos x)}(\sin^2 x - \sin x \cos x + \cos^2 x)}{\cancel{\sin x + \cos x}}$$

$$= \sin^2 x - \sin x \cos x + \cos^2 x$$

$$= \boxed{1 - \sin x \cos x}$$

Oct 3-10:43 AM

$$\sin^2 x + \cos^2 x = 1$$

$$\tan x = \frac{\sin x}{\cos x} \qquad \cot x = \frac{\cos x}{\sin x} \qquad \cot x = \frac{1}{\tan x}$$

$$\csc x = \frac{1}{\sin x} \qquad \sec x = \frac{1}{\cos x}$$

$$1 + \tan^2 x = \sec^2 x \qquad 1 + \cot^2 x = \csc^2 x$$

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$\sin(A-B) = \sin A \cos B - \cos A \sin B$$

$$\cos(A-B) = \cos A \cos B + \sin A \sin B$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

Oct 3-10:48 AM

Verify

$$\boxed{\sin(x+y)} - \boxed{\sin(x-y)} = 2\cos x \sin y$$

$$\sin x \cos y + \cos x \sin y - (\sin x \cos y - \cos x \sin y) =$$

$$\cancel{\sin x \cos y} + \cos x \sin y - \cancel{\sin x \cos y} + \cos x \sin y =$$

$$2\cos x \sin y$$

Oct 3-10:54 AM

Verify $\tan\left(x - \frac{\pi}{4}\right) = \dfrac{\tan x - 1}{\tan x + 1}$

$\frac{\pi}{4} = 45^\circ$

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

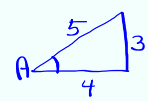
$\tan 45^\circ = 1$

$$\tan\left(x - \frac{\pi}{4}\right) = \frac{\tan x - \tan\frac{\pi}{4}}{1 + \tan x \cdot \tan\frac{\pi}{4}} \qquad \frac{\tan x - 1}{1 + \tan x \cdot 1}$$

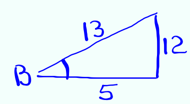
$$= \frac{\tan x - 1}{\tan x + 1}$$

Oct 3-10:58 AM

$0^\circ < A < 90^\circ \qquad \sin A = \frac{3}{5}$



$0^\circ < B < 90^\circ \qquad \cos B = \frac{5}{13}$



$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$
$$= \frac{3}{5} \cdot \frac{5}{13} + \frac{4}{5} \cdot \frac{12}{13} = \frac{15}{65} + \frac{48}{65} = \boxed{\frac{63}{65}}$$

$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$
$$= \frac{4}{5} \cdot \frac{5}{13} + \frac{3}{5} \cdot \frac{12}{13} = \frac{20}{65} + \frac{36}{65} = \boxed{\frac{56}{65}}$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} = \frac{\frac{3}{4} + \frac{12}{5}}{1 - \frac{3}{4} \cdot \frac{12}{5}}$$

LCD = 20

$$= \frac{\cancel{20}^{5} \cdot \frac{3}{\cancel{4}} + \cancel{20}^{4} \cdot \frac{12}{\cancel{5}}}{20 \cdot 1 - \cancel{20} \cdot \frac{3}{\cancel{4}} \cdot \frac{12}{\cancel{5}}}$$

$$= \frac{15 + 48}{20 - 36} = \frac{63}{-16} = \boxed{-\frac{63}{16}}$$

Oct 3-11:04 AM

$\sin(A+B) = \sin A \cos B + \cos A \sin B$

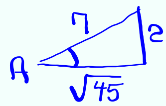
Replace B with A, simplify

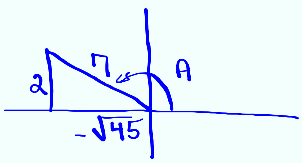
$\sin(A+A) = \sin A \cos A + \cos A \sin A$

$\boxed{\sin 2A = 2 \sin A \cos A}$ Double-Angle formula

ex: $90^\circ < A < 180^\circ$ $\sin A = \frac{2}{7}$ Find $\sin 2A$

$180^\circ < 2A < 360^\circ$ $= -$





$\sin 2A = 2 \sin A \cos A$

$= 2 \cdot \frac{2}{7} \cdot \frac{-\sqrt{45}}{7}$

$\sqrt{45} = \sqrt{9}\sqrt{5} = 3\sqrt{5}$

$= \frac{-4\sqrt{45}}{49} = \boxed{\frac{-12\sqrt{5}}{49}}$

Oct 3-11:14 AM

$\cos(A+B) = \cos A \cos B - \sin A \sin B$

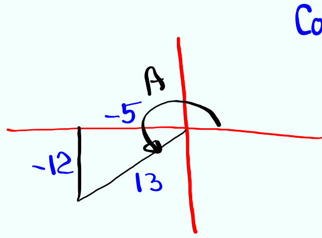
Replace B with A, simplify

$\cos(A+A) = \cos A \cos A - \sin A \sin A$

$\boxed{\cos(2A) = \cos^2 A - \sin^2 A}$ Double-Angle formula

$180^\circ < A < 270^\circ$ $\cos A = \frac{-5}{13}$ Find $\cos 2A$

QIII



$\cos 2A = \cos^2 A - \sin^2 A$

$= \left(\frac{-5}{13}\right)^2 - \left(\frac{-12}{13}\right)^2$

$= \frac{25}{169} - \frac{144}{169} = \frac{-119}{169}$

Oct 3-11:22 AM

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

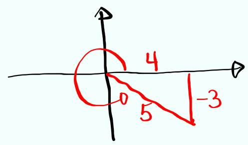
Replace B with A, Simplify

$$\tan(A+A) = \frac{\tan A + \tan A}{1 - \tan A \tan A}$$

$$\boxed{\tan 2A = \frac{2\tan A}{1 - \tan^2 A}}$$ Double-Angle Formula

$270° < A < 360°$ $\quad \tan A = \frac{-3}{4}$ $\quad$ Find $\tan 2A$

QIV



$$\tan 2A = \frac{2\tan A}{1-\tan^2 A} = \frac{\cancel{2}\cdot\frac{-3}{\cancel{4}_2}}{1-\left(\frac{-3}{4}\right)^2}$$

$$= \frac{\frac{-3}{2}}{1-\frac{9}{16}} = \frac{\overset{8}{\cancel{16}}\cdot\frac{-3}{\cancel{2}}}{16\cdot 1-\cancel{16}\cdot\frac{9}{\cancel{16}}} = \frac{-24}{16-9}$$

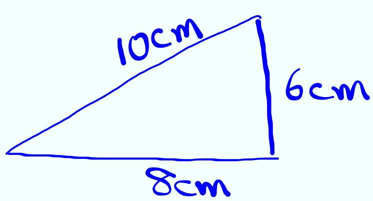
LCD=16 $\quad = \boxed{\frac{-24}{7}}$

Oct 3-11:28 AM

Class Quiz 6

Use Heron's Formula to find the area of the triangle below

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$



$$s = \frac{a+b+c}{2}$$

$$s = \frac{6+8+10}{2} = \frac{24}{2} = 12$$

$$\text{Area} = \sqrt{12(12-6)(12-8)(12-10)}$$

$$= \sqrt{12\cdot 6\cdot 4\cdot 2}$$

$$= \sqrt{576} = \boxed{24\text{ cm}^2}$$

Oct 3-11:35 AM